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LETTER TO THE EDITOR

Equipartition of current in parallel conductors on cooling through the superconducting transition

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Abstract

Our experiments show that for two or more pieces of a wire, of different lengths in general, combined in parallel and connected to a dc source, the current ratio evolves towards unity as the combination is cooled to the superconducting transition temperature T_c , and remains pinned at that value below it. This redistribution of the total current towards equipartition without external fine-tuning is a surprise. It can be physically understood in terms of a mechanism that involves the flux-flow resistance associated with the transport current in a wire of type-II superconducting material. It is a fact that the flux-flow resistance increases with the current that drives the current division towards equipartition.

In the course of our experiments, with the original objective of settling the now not-so-frequently asked, but nevertheless askable question [1] as to whether the resistance in the superconducting state is absolutely zero, or merely too small, we were led to investigating how a given dc current divides between two or more conductors of different resistances connected in parallel as the combination is cooled through their common critical temperature T_c . To our surprise, the current ratio evolves monotonically, with cooling, towards unity, and stays pinned at that value below T_c . This equipartition of the total transport current between the two or more conducting arms connected in parallel, on approaching T_c , like also its persistence below T_c , does not follow from our conventional understanding of the macroscopic current transport through the normal or the superconducting circuits [2–4], to the best of our knowledge. After all, while the two resistances in parallel must go to zero simultaneously at their common transition temperature, their ratio could be arbitrarily different from unity.

The fact that the current ratio evolves from its usual ohmic value (far from unity in the normal state much above T_c) to a ‘fixed point’ value ≈ 1 as $T \rightarrow T_c$, and remains pinned at this

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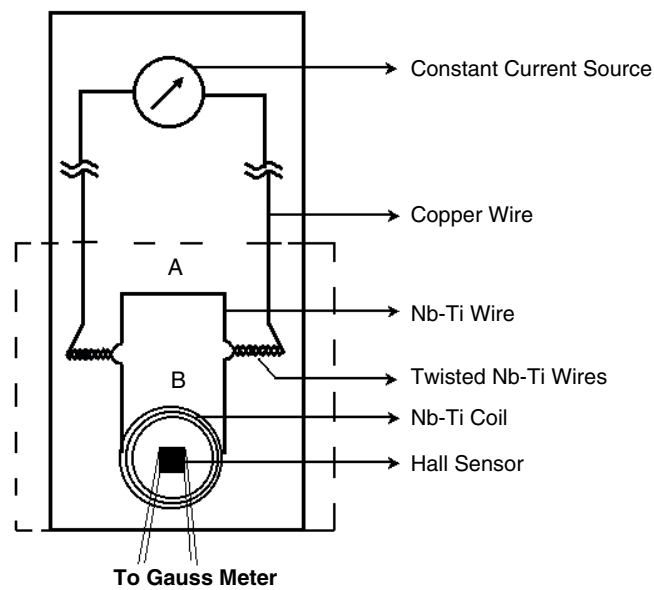


Figure 1. The sketch of the experimental set-up used. The part of the assembly enclosed in the dashed box could be inserted in an Oxford Instruments continuous flow cryostat for temperature variation. The coil B of diameter 20 mm has 40 turns of NbTi wire. Path A is made up of about 6 cm straight length of the same NbTi wire. The respective ends of the two paths are twisted together for about 2.5 cm each so that a single NbTi path connects the NbTi loop to the copper wire on either side. This is to eliminate any possible effects of asymmetric bifurcation of the current and differential terminal resistance between the superconducting loop and the normal copper wire.

value in the superconducting state, raises several basic questions, in addition to being of obvious relevance to superconducting circuits and electronics. Below, we describe our experiment that uses contactless current measurement (to avoid perturbing the current ratio). This is followed by a discussion of our main result, namely that of equipartition in physical terms of a mechanism involving the flux-flow resistances associated with the transport currents through the conductors connected in parallel.

The experimental arrangement used by us is sketched in figure 1. Here A and B form the two parallel arms of a superconducting loop of NbTi superconducting wire. A is a straight wire of length l_A (about 6 cm in one of our experiments) and B, of length l_B (~ 260 cm in our experiments), is in the form of a circular coil in the proximity of which a precision Hall probe sensor S is placed for contactless current measurements. To start with, path A is disconnected and a current I ($=6$ A in a typical experiment) from a constant current source is passed through path B. This generates a magnetic field H_{n1} whose value ($=132$ G) is measured by the Hall probe gaussmeter. This contactless probe ensures that the measurement itself does not perturb the current division in any way. Then the path A is connected in parallel with B. (While doing this the respective ends of the two wire pieces are twisted together for about 2.5 cm each so that a single NbTi path connects the NbTi loop to the copper wire on either side. This is to eliminate the effects of any possible asymmetric bifurcation of the current and the consequent differential terminal resistance when the loop goes superconducting.) Now the field H_{n2} generated by B is again measured. It is found that $H_{n2} = 3$ G which is nothing but $H_{n1}/(k+1)$, where $k = l_B/l_A$ is the ratio of the two wire lengths or equivalently of their resistances. This is of course, a simple consequence of the fact that the two resistances are connected in parallel.

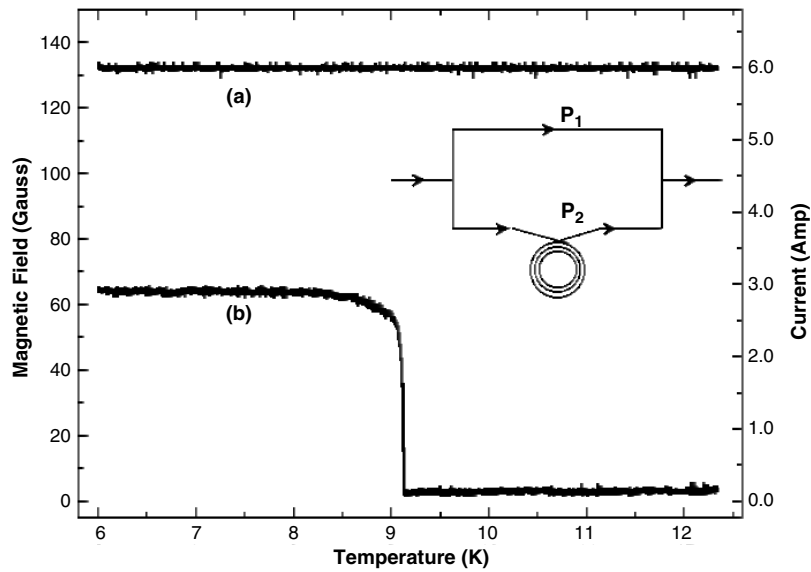


Figure 2. Equipartition in two parallel superconducting paths. The magnetic field measured and the corresponding current flowing through the coil are shown (a) when the entire current is flowing through path P_2 only and (b) when the path P_1 is connected in parallel with P_2 . The inset shows a schematic diagram of the experimental arrangement.

Now the assembly is cooled to liquid helium temperature (4.2 K), i.e., below the transition temperature of the NbTi wire ($T_c = 9.3$ K), and the magnetic field produced by the coil B is measured again. We find a value of 65 G corresponding to a current flow of 3 A, i.e. half of the total current. Figure 2 presents the results of this experiment along with the schematic diagram in the inset. In figure 3, the results of another experiment (carried out with a different coil) are also shown in which we had three parallel paths instead of two. In this case, we observed that a third of the total current passed through the coil. Thus the main observation of this work is that the current ratio evolves from its usual ohmic value (far from unity in the present case) in the normal state much above T_c , to a value very close to unity as the transition (which had a small but finite width) was approached from above T_c . Moreover, the current ratio remained pinned at this value (i.e. close to unity) in the superconducting state below T_c . This evolution of the resistance ratio (or equivalently, the current ratio) converging to a ‘fixed point’ ≈ 1 , together with its remaining pinned at this value below T_c , seems to have escaped attention until now. This evolution of the current ratio towards equipartition in the transition region clearly involves some interesting physics—of a redistribution of the total current towards equality between the parallel arms without requiring any fine-tuning. In the following a possible interpretation of this result is given in the physical terms of a mechanism involving the flux-flow resistance of a current carrying wire of the type-II superconducting material.

Consider first the flux-flow resistance of a type-II superconducting wire carrying a transport current. As is well known, the axial current generates an azimuthal magnetic field, and the corresponding flux is quantized into toroidal flux tubes linking the axial current. The latter exerts a Lorentz force causing the flux tubes to move radially. This radial motion of the flux tubes in turn induces an electric field along the axial transport current giving rise to dissipation, and hence equivalently to a resistance—the flux-flow resistance. This flux-flow resistance (dissipation), however, requires depinning of the flux tubes. Inasmuch as the

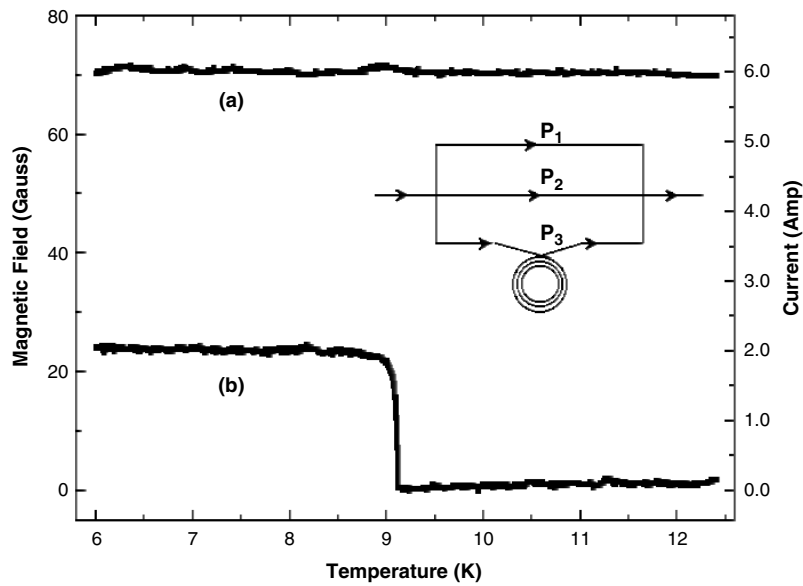


Figure 3. Equipartition in three parallel superconducting paths. The magnetic field measured and the corresponding current flowing through the coil are shown (a) when the entire current is flowing through the coil and (b) when two separate wires are connected in parallel with the coil.

depinning force is an increasing function of the transport current, the corresponding resistance increases with the latter. Thus, for two such conductors connected in parallel and carrying currents I_1 and I_2 with $I_1 + I_2 = I$, the impressed total current from a current source, we have $\partial R_1/\partial I_1 > 0$, $\partial R_2/\partial I_2 > 0$, implying a non-linearity, namely that the conductor initially carrying a higher current will have its resistance relatively raised, tending to suppress the current. This suppression provides a non-linear negative feedback that drives the currents in the two arms towards equality (equipartition) iteratively.

While the above flux-flow mechanism providing a resistive non-linear negative feedback is general, it is expected to iterate towards equipartition only for the experimental situation considered here, namely, where the system of conductors in parallel is already carrying the total impressed current I and is then cooled down towards the critical temperature, i.e. it is in the transition regime. (Indeed, the situation where the system is already in the superconducting state and then the current is impressed is quite different. Here we should expect to observe the usual relation $I_1 L_1 = I_2 L_2$ to hold [2], where L_1 and L_2 are the inductances of the two arms. This was indeed verified by us to be the case.)

The reason for this can be appreciated from the following considerations. The transition region ($T \geq T_c$) is dominated by the fluctuation superconductivity [5], where the pinning is relatively weak and the current induced depinning effectively controls the flux-flow resistance. Thus $\partial R_i/\partial I_i > 0$ (where 'i' labels the branches in parallel) together with the fact that $R_i \cong 0$ as $T \rightarrow T_c$ drives the current division towards equipartition. Of course, once below T_c the equipartition is protected through the superconductive diamagnetism (flux expulsion). A simple formal implementation of the above negative feedback due to the current dependent flux-flow resistance supports the above physical picture. Indeed, it is interesting that a straightforward application of the extremal dissipation principle⁴ [7], i.e., extremizing the total dissipation

⁴ For a recent discussion of generalization of the Onsager–Rayleigh principle of least dissipation of energy, see Dewar [6].

with respect to the transport currents I_i subject to the constraint $\sum_i I_i = I = \text{constant}$ and $\partial R_i / \partial I_i > 0$ gives $\partial \ln R_1 / \partial \ln I_1 = \partial \ln R_2 / \partial \ln I_2 = \dots$. The latter implies $I_1 = I_2 = \dots$ in the limit $R_i \rightarrow 0$ (i.e. the approach to T_c) under the physical condition of monotonic current dependence of the logarithmic derivatives [8].

Clearly, the interesting question as to how general the above conclusion is requires further work, e.g., trying different diameters and materials for the two resistances in parallel, but, of course, these must have the same critical temperature. An interesting system to study would be the case of wires of weakly linked granular materials, e.g. the high T_c superconducting ceramic materials, where the transport current induced suppression of the superconducting weak links in the wires is well described by the Lawrence–Doniach model [9], and should provide the non-linear negative feedback discussed above. Moreover, here the transition is much broader, making it easier to probe the evolution towards equipartition experimentally.

In summary, we have reported a novel phenomenon with respect to the division of current between parallel conducting paths in which the total current is found to divide equally among the paths (irrespective of their inductances and the initial normal-state resistances) as the system is cooled to and below the common superconducting transition at T_c . This equipartition should be relevant to system geometries where the divided (series–parallel) superconducting circuit is cooled to and below T_c in the presence of a transport current. It also provides an interesting laboratory example for self-organization without fine-tuning, and possibly for the application of the principle of extremal dissipation, that has been attracting considerable attention lately [6, 7].

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